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Dynamic behaviour of a boundary layer with condensation along a flat plate: comparison with suction

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Abstract—The dynamical behaviour of laminar and turbulent boundary layers inside which a condensation phenomena exists, has been experimentally and numerically studied. The temperature difference between the exchange cold wall and the saturated air–steam flow at atmospheric pressure and moderate temperature (~80–90°C) does not exceed 20°C. Evaluation of the mass flux at the wall in such a case, allows the prediction of the 'equivalent suction rate' in a sucked boundary layer for the same external conditions but without any temperature difference. The comparison between the two phenomena reveals interesting similarities and some differences.

1. INTRODUCTION

THE AIM of the present work is to study the behaviour of a laminar or a turbulent boundary layer inside which exists a condensation phenomena due to the temperature difference between the wall and the main flow of a saturated air–steam mixture. Since the dynamic behaviour of such a boundary layer is similar to the behaviour of sucked layers, it was found useful to compare the numerical results of these two problems.

During the past 30 years, boundary layers with suction have been studied by a few authors [1–4]. For laminar and turbulent boundary layers, the film theory enables a simplified analytical treatment both for momentum, energy and diffusion equations [1]. For the turbulent boundary layer, some logarithmic or bilogarithmic laws of the wall were used and experimental results compared with these theoretical approaches [2, 3]. Moreover, recent investigations addressed condensation phenomena inside a boundary layer or only at the wall in forced convection flows of gas–vapour mixtures. Most authors assumed that phase change takes place only on the condensate film [5, 6]; the present work, like [7], takes into account the relationship between concentration and temperature at each point. This relation ensures thermal equilibrium in the whole field by partial condensation in order to maintain everywhere a saturated air–steam mixture. This thermal equilibrium condition has to be added to the mass, momentum and energy conservation equations. No hypothesis is made on the distribution profiles of the variables. They are determined step-by-step by a finite-differences method [8]. An integral calculation method has also been used [9]; it gives results in good agreement with the present ones.

In this study, the condensation problem will be first exposed: numerical results will be compared with experiments obtained in a wind tunnel over a flat plate. Calculation is then applied to suction; for this purpose, it is necessary to determine an 'equivalent suction rate' and to define precisely the meaning of 'equivalent'. Finally, dynamic results for laminar and turbulent boundary layers will be compared with the corresponding sucked layers.

2. THEORETICAL ANALYSIS

The model used for the analysis of the problem is illustrated in Fig. 1. In order to use the classical equations of conservation in such a boundary layer, it is necessary to make the following hypotheses:

- The liquid film on the wall is so thin that it does not disturb the boundary layer and it does not sensibly modify the heat transfer.
- The volume of the condensate droplets is negligible; there is no interaction between two droplets; the droplet velocity is equal to the gas-phase velocity $\vec{u}$.
- Longitudinal diffusion velocity is very small when compared to the velocity u.
- Gravity and natural convection are neglected.

The basic equations are the Navier–Stokes, the energy and the diffusion equations

$$\left.\begin{aligned}
&\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \\
&\frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \\
&\frac{\partial}{\partial x}(\rho hu) + \frac{\partial}{\partial y}(\rho hv) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) \\
&\qquad + \frac{\partial}{\partial y}\left\{(\rho_g + \rho_v) D (h_g - h_v)\, \partial\left(\frac{\rho_g}{\rho_g + \rho_v}\right)\Big/ \partial y\right\} \\
&\frac{\partial}{\partial x}(\rho_g u) + \frac{\partial}{\partial y}(\rho_g v) = \frac{\partial}{\partial y}\left\{(\rho_g + \rho_v) D \frac{\partial}{\partial y}\left(\frac{\rho_g}{\rho_g + \rho_v}\right)\right\}
\end{aligned}\right\}$$

NOMENCLATURE

C_f skin friction coefficient
c_p specific heat at constant pressure [J kg^{-1} K^{-1}]
D diffusion coefficient [m^2 s^{-1}]
h enthalpy [J kg^{-1}]
L latent heat of condensation [J kg^{-1}]
p pressure
T temperature [K]
ΔT temperature difference between the main flow and the wall
u, v velocity components [m s^{-1}]
u_τ friction velocity [m s^{-1}]
x, y coordinates [m].

Greek symbols
δ boundary-layer thickness [m]
λ thermal conductivity [W m^{-1} K^{-1}]
μ dynamic viscosity [kg m^{-1} s^{-1}]
ν kinematic viscosity [m^2 s^{-1}]
ρ mass per unit volume of mixture [kg m^{-3}]
ψ streamfunction.

Subscripts
g non-condensable gas
l liquid
v vapour
w wall
∞ main flow.

where

$$\rho u = \rho_g u_g + \rho_v u_v + \rho_l u_l$$
$$\rho v = \rho_g v_g + \rho_v v_v + \rho_l v_l$$
$$\rho h u = \rho_g h_g u_g + \rho_v h_v u_v + \rho_l h_l u_l$$

and similarly for $\rho h v$, with

$$h_g = c_{pg}(T - T_\infty), \quad h_v = c_{pv}(T - T_\infty),$$
$$h_l = c_{pl}(T - T_\infty) - L.$$

To these four equations, the thermal equilibrium relation must be added $p_v = f(T)$. When a constant static pressure in all the boundary layer is assumed, the air partial pressure can be known, and then, if the two gases are assumed to be perfect, the mass concentrations of air and steam can be deduced from the local temperature value T. Hence, in this set of equations, only four unknowns remain: u, v, T and ρ_l [10].

It must be noted that this set of equations is valid for both laminar and turbulent boundary layers when a laminar or 'effective' viscosity is introduced.

The boundary conditions at the wall are:

$$u = 0$$
$$v_g = 0$$

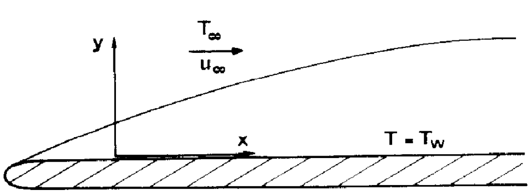

FIG. 1. Schematic configuration.

and, hence, from Fick's law:

$$v_w = \left[\frac{\rho_g + \rho_v}{\rho_g} D \frac{\partial}{\partial y}\left(\frac{\rho_g}{\rho_g + \rho_v}\right)\right]_w$$
$$T = T_w$$
$$\rho_l = \rho_{lw};$$

for laminar boundary layer, Hijikata and Mori's ρ_{lw} value is used [7]; for the turbulent boundary layer, the ρ_{lw} value is obtained by successive runs until the profiles of velocity, temperature and concentration remain smooth in the whole field.

In the main flow, we have $u = u_\infty$, $T = T_\infty$, $\rho_l = 0$.

For the effective viscosity, Prandtl's mixing length hypothesis has been used with the polynomial approximation due to Pletcher [11]. When mass transfer occurs at the wall, as happens with condensation, the constant of the Van Driest damping factor must be modified since it depends upon the transversal velocity at the wall v_w. These modifications have already been studied for air only [12] but, to our knowledge, not for mixed gases. If it is assumed that the same modifications can be done for mixed gases, a modified damping factor and so a new mixing length are obtained. However, for the small temperature differences ($< 20°$C) examined in this paper, the v_w value is very small ($< 8 \times 10^{-3}$ m s^{-1} for $T_\infty = 85°$C and $\Delta T = 15°$C), and hence, the damping factor and the mixing length are very close to their values without any suction. Similarly, the effective Prandtl and Schmidt numbers can be assumed constant in the whole boundary layer and equal to their classical values for air.

3. NUMERICAL TREATMENT

When the streamfunction ψ is used, three of the four partial differential equations remain with three unknowns u, T, ρ_1. As they are coupled, no analytical solution can be found and they must be numerically integrated. A finite-difference method, which is an adaptation of the general Patankar and Spalding's treatment [8] to the condensation case, enables us to transform the partial differential equations into algebraic ones. When profiles of the three functions are known at the abscissa $x-\Delta x$, the new values are calculated, at the abscissa x, at any point of the grid covering the boundary layer. Then, if the physical properties of each component of the mixture are known and the conditions of the stream (pressure, velocity and temperature) are imposed, the calculation can yield, for each wall temperature and with an imposed value of ρ_{1w}, the dynamic, thermal and concentration fields in the whole boundary layer.

Application to suction

In order to compare condensation and suction, the previous calculations were performed for the same mixing stream without temperature difference but with an imposed suction velocity at the wall. The problem consists in the determination of the suction rate which will give a boundary layer of the same dynamic thickness and with a transverse velocity profile close to the one obtained in the condensation case. It should be noted that the suction rate imposed at the wall remains constant along the whole plate, although, in condensation, the mass flux at the wall is a decreasing function of x. Hence, for a given suction rate, the agreement is quite good for the beginning of the plate, but not so good further downstream. This is the reason why it is more convenient to prescribe the suction rate which is attained far away on the plate and to compare two profiles of this region.

4. EXPERIMENTAL PROCEDURE

Experiments were carried out in a low-speed wind tunnel ($u_\infty < 6$ m s^{-1}) at atmospheric pressure. The main devices are a 108 kW electrical air heater, a 5000 m^3 h^{-1} air compressor and a 1500 kg h^{-1} steam generator which supplies an air–steam mixture at the desired relative humidity. For this study, the saturation condition is imposed and is verified during each run by measuring the amount of condensed water in a given time. In order to avoid undesirable condensation on the walls, they are electrically heated and thermally insulated. The test chamber is 0.8 m long and 0.2×0.4 m^2 cross-section. The condensing surface is the bottom of the test chamber itself; it is a brass flat plate heated at a uniform temperature T_w, lower than the main flow one T_∞. The temperature differences do not exceed 20°C because larger values of ΔT induce fog formation near the walls and make dynamic and thermal measurements much more difficult.

Two kinds of boundary layers were examined: laminar and turbulent. For the laminar one, an elliptic leading edge has been put at the beginning of the flat plate and, upstream, a suction slit was made. For the turbulent boundary layer, the slit has been shut and the turbulence was caused by tripping wires in order to obtain a fully-developed turbulent boundary layer which can be correctly compared with the numerical calculations. This fully-developed character of the turbulent boundary layer was verified by systematic explorations of the boundary layer with a total pressure probe which could be moved along two perpendicular directions. Unfortunately, no measurement of the transverse velocity could be performed so far; however, the good agreement between the observed and calculated velocity profiles enabled us to present the calculated transversal velocity profiles.

5. RESULTS: DYNAMICAL ASPECT

In laminar and in turbulent boundary layers, the condensation and the suction phenomena were compared only from a dynamical point of view. The flow conditions are the same for both cases; however, in order to obtain condensation, the wall must be colder than the flow whereas, when suction is studied, the wall temperature is made equal to T_∞.

5.1. *Laminar boundary layer*

In the Figs. 2–5, various cases at the wall are represented for the same conditions of steam–air flow. Figure 2 shows velocity profiles at the abscissa $x =$ 0.7 m; it may be observed that the condensation as well as the suction at the wall induces a reduction of the boundary-layer thickness and that a rate of suction exists for which the two phenomena yield the same velocity profile. For condensation, experimental points are plotted on this figure; they are in good agreement with the calculated ones.

Transversal velocity profiles are shown in Fig. 3 in the same cases as in Fig. 2. A small discrepancy exists between condensation (curve 3) and suction (curve 4). For this abscissa, a smaller suction rate should be more adequate to nearly obtain the same transversal velocity profile. From an inspection of this figure, it is obvious that the larger the suction rate is, the more important is the sucked part of the boundary layer. For example, curve 1 which exhibits the case without suction, is always positive whereas, in curve 2, the sucked part (negative transversal velocity region) nearly reaches the boundary-layer thickness.

In Fig. 4, the dynamical boundary-layer thickness development is plotted vs the abscissa x in the various cases previously examined. A rate of suction equal to -2×10^{-3} kg m^{-2} s^{-1} still gives a good agreement with the condensation. In the lower part of this figure, the sucked portion of the boundary layer is obviously represented in the two cases 3 and 4. It is clear that the suction condition at the wall induces a more important increase of this sucked region than condensation will do. Figure 5 shows the variation of the local skin friction coefficient vs the Reynolds number Re_x, in the same four cases. The straight line of curve 1 also corresponds to the Blasius theory. The two cases of

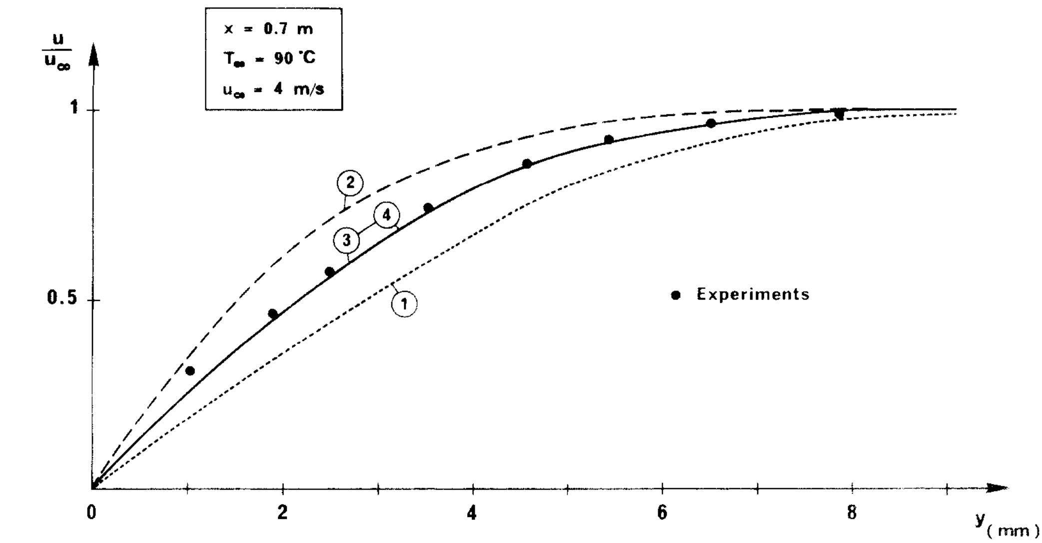

FIG. 2. Laminar velocity profiles: 1, without temperature difference and without suction; 2, suction at a rate of $\rho_w v_w = -5 \times 10^{-3}$ kg m^{-2} s^{-1}; 3, condensation with $\Delta T = 10$°C; 4, suction at a rate of $\rho_w v_w = -2 \times 10^{-3}$ kg m^{-2} s^{-1}.

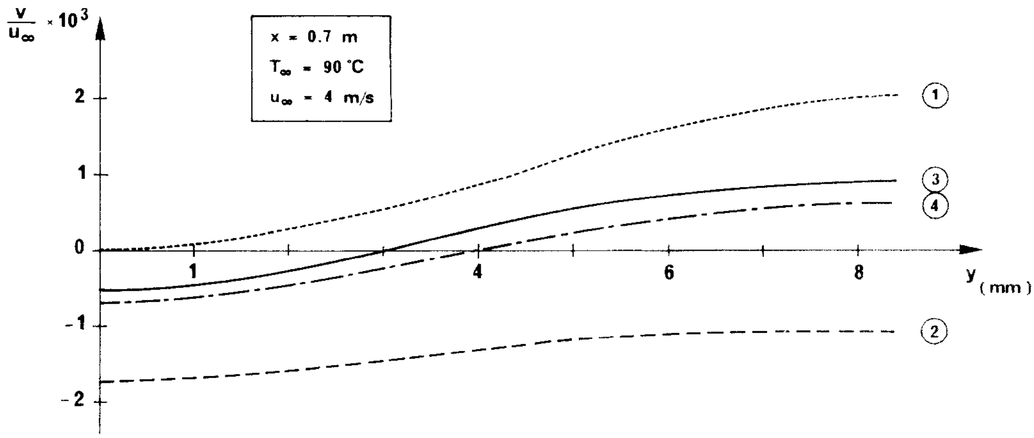

FIG. 3. Transverse velocity profiles in laminar boundary layer (same lengend as in Fig. 2).

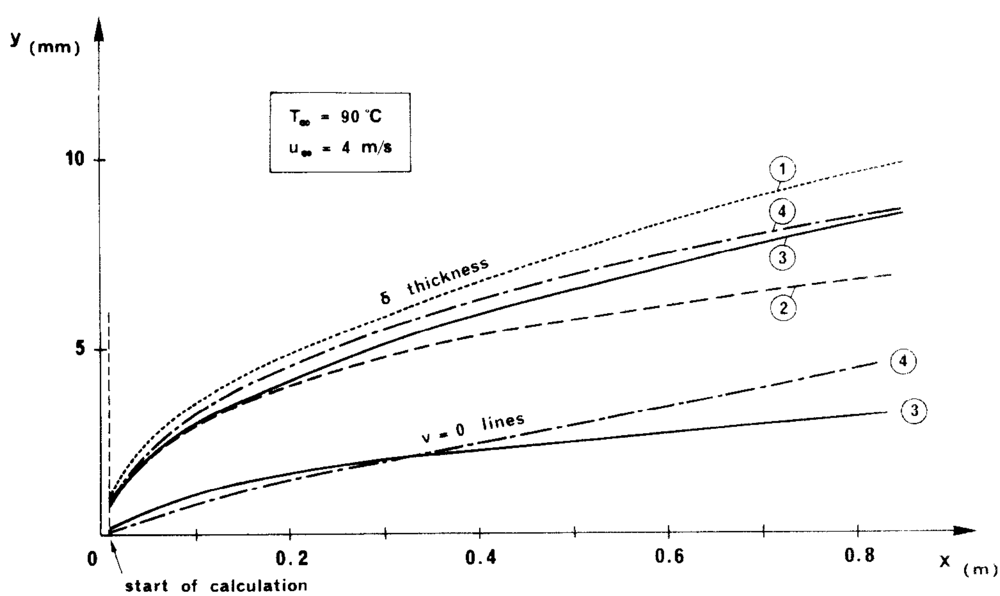

FIG. 4. Boundary-layer thicknesses and $v = 0$ lines (laminar) (same legend as in Fig. 2).

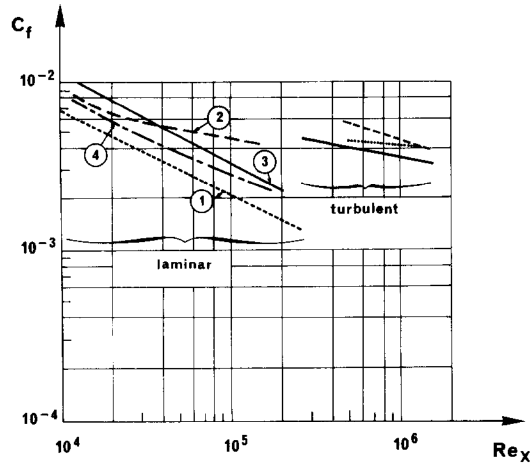

FIG. 5. Laminar (same legend as in Fig. 2). Turbulent, $T_\infty = 85°C$: ---- condensation $\Delta T = 15°C$; ···· suction $\rho_w v_w = -0.44 \times 10^{-2}$ kg m^{-2} s^{-1}; —— Prandtl theory.

suction 2 and 4 are convex, the upper curve corresponding to the more important suction rate; such a variation has been already found by Schlichting [13]. The difference between condensation (curve 3) and suction is the most visible in this figure. The 'equivalent' suction induces a coefficient of skin friction sensibly lower than the condensation will do.

5.2. *Turbulent boundary layer*

As previously, experiments were compared with the numerical results by means of the velocity profiles. In order to do so, it was necessary, as already stated, to obtain in the wind tunnel, a fully-developed turbulent boundary layer. For a flat plate and for air only, such a profile follows the universal law:

$$\frac{u}{u_\tau} = A \log \frac{y u_\tau}{\nu} + B$$

where A and B are empirical constants and u_τ is the friction velocity at the wall. From this expression, we deduce:

$$\frac{u}{u_\infty} = A' \log \frac{y u_\infty}{\nu} + B'$$

where

$$\frac{A'}{A} = \frac{u_\tau}{u_\infty}.$$

From the plot of the experimental profile of u/u_∞ vs yu_∞/ν in semi-logarithmic coordinates, the value of u_τ can be easily determined; since the constant A is known, the u_τ value is deduced from the slope of the straight portion of this profile.

Figure 6 gives two examples of such a determination of the u_τ value from experiments for saturated air–steam flows with and without heat transfer. It should be noted that, as for laminar boundary layer, u_τ and hence C_f increase with condensation. When u_τ is deduced in this way from the data, it becomes possible to compare calculated and measured velocity profiles, as in Fig. 7. The agreement is good indeed because, as a matter of fact, for moderate flow temperatures (~85°C) and small temperature differences (less than 20°C), the equivalent suction rate $-v_w/u_\infty$ is lower than 9×10^{-4},

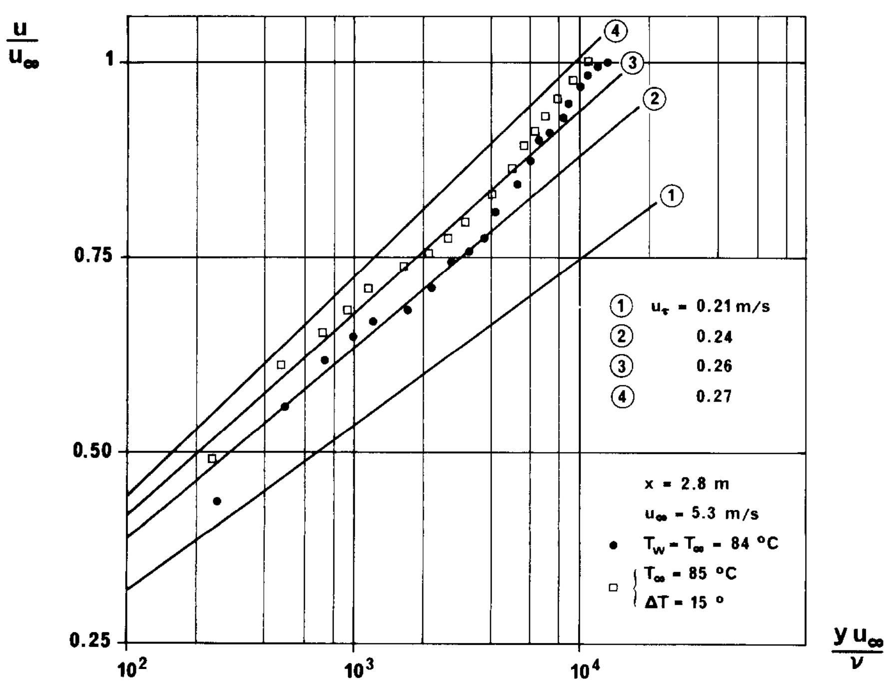

FIG. 6. Experimental determination of u_τ.

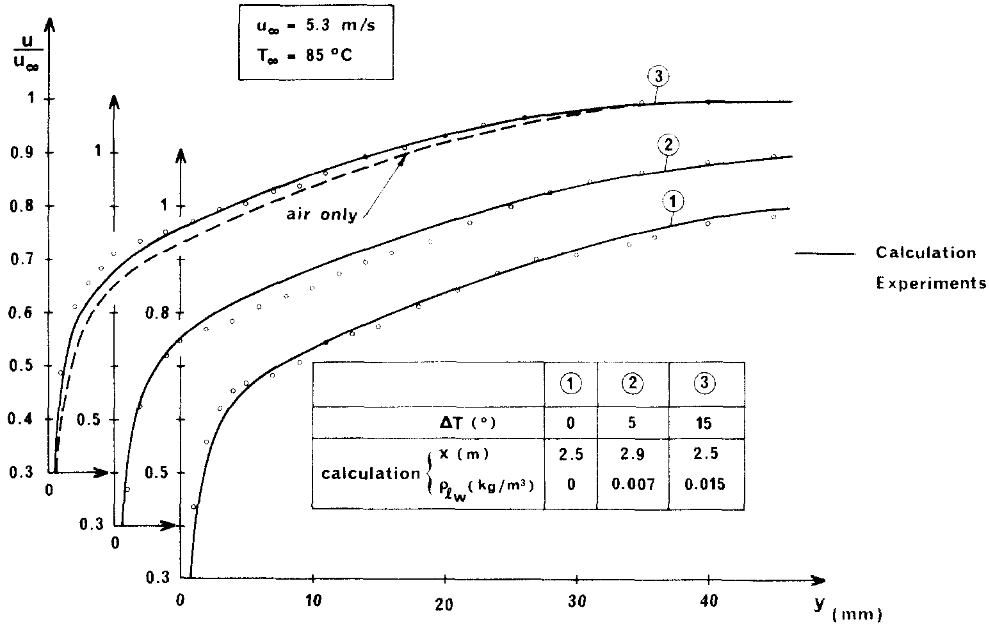

FIG. 7. Turbulent velocity profiles.

and then the velocity profile displays a very slight distortion [3]. In the calculation, the abscissa x is fitted by comparison between the calculated and the experimental boundary-layer thicknesses.

In Figure 8, are shown the transverse velocity profiles calculated from the streamfunction ψ for the three experimental cases of Fig. 7. The larger the temperature difference is, the more important the negative portion of the transverse velocity profile is, until it fills up nearly the whole boundary layer for $\Delta T = 15°C$. In this figure, the calculated transverse velocity profiles obtained in two cases of sucked boundary layer for the same external conditions of flow are also plotted. For a suction rate at the wall $\rho_w v_w = -0.44 \times 10^{-2}$ kg m^{-2} s^{-1}, the discrepancy with the condensation case of $\Delta T = 15°C$ is about 10% whereas the u/u_∞ profile calculated for the same rate of suction is exactly superimposed on the curve 3 of Fig. 7 (discrepancy equal to 1%).

As in Fig. 4, the development of the boundary-layer thickness along the plate is represented in Fig. 9, for $T_\infty = 85°C$ and two temperature differences. It should be noted that, for $T_\infty = 75°C$ and $\Delta T_\infty = 15°C$, the curves are nearly the same as for $T_\infty = 85°C$ and $\Delta T_\infty = 5°C$; they correspond to a closed suction rate. As expected, the boundary-layer thickness increases when ΔT decreases; this is due to the sucking effect of condensation. The more important the condensation is, the greater the suction is. In these two cases of condensation, the equivalent suction phenomena has

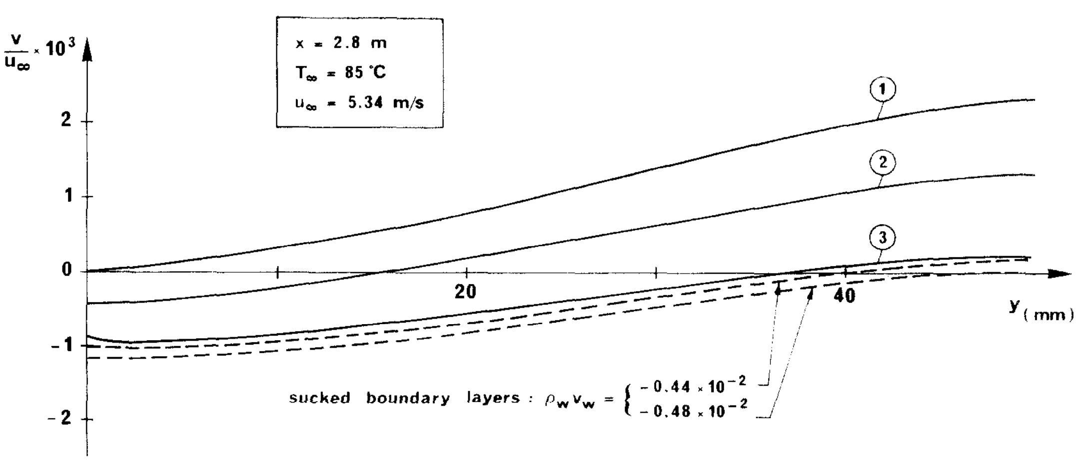

FIG. 8. Transverse velocity profiles in turbulent boundary layer.

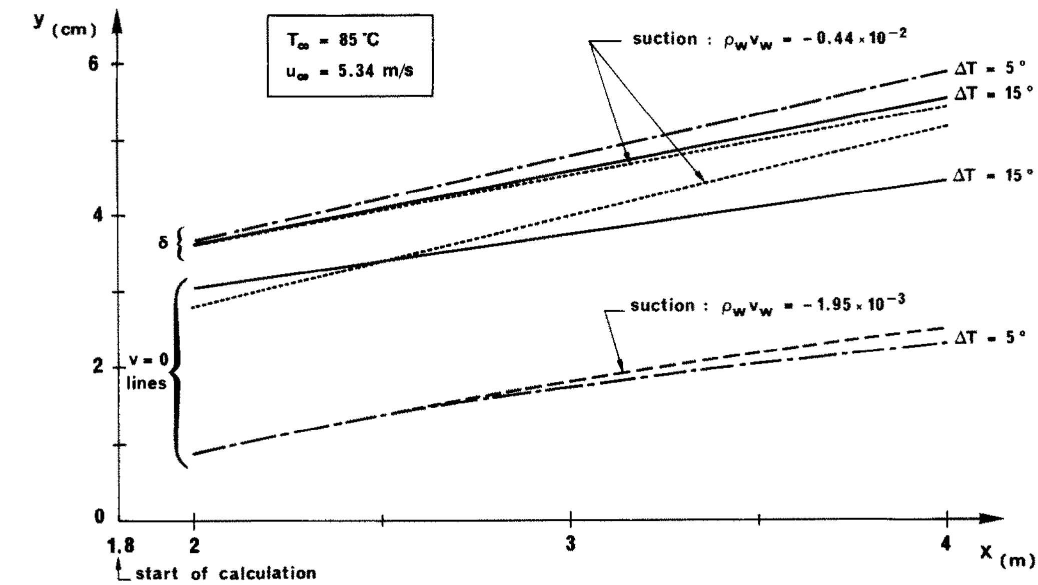

FIG. 9. Boundary-layer thicknesses and $v = 0$ lines (turbulent).

nearly exactly the same δ evolution along x. In this figure, are also plotted the $v = 0$ lines which separate the boundary layer into two parts: near the wall, the sucked region where v is lower than 0, and the external spreading out region. It should be noted that, when $\Delta T = 5°C$, the sucked part of the boundary layer fills only 1/3 of the total thickness whereas, for $\Delta T = 15°C$ it fills more than 3/4. When the condensation rate is low, the boundary-layer thickness evolution is nearly the same for sucked or condensed flows; but, if ΔT is greater, the discrepancy increases and the region where v is positive is smaller and smaller as x increases for the suction phenomena whereas it grows with x for the condensation one.

As previously noted in the laminar boundary layer, this is, with the drag coefficient, the most important difference between the two phenomena. The drag coefficient is shown, for the turbulent case, in the upper-right part of Fig. 5. The effect of suction is to lower the drag coefficient in comparison with condensation but it remains higher than the values of Prandtl's theory.

6. CONCLUSIONS

As it has been shown in this study, from a dynamical point of view, the boundary layer along a flat plate when condensation takes place in presence of an incondensable gas, is similar to the one with suction at the wall. The equivalent suction rate has been determined from the results of the mass flux at the wall in the condensation case. Only a small difference appears in the skin friction and in the part of boundary layer where the transverse velocity is negative.

Heat and mass transfer with condensation was not addressed here but is postponed to a subsequent paper.

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COMPORTEMENT DYNAMIQUE D'UNE COUCHE LIMITE AVEC CONDENSATION LE LONG D'UNE PLAQUE PLANE: COMPARAISON AVEC L'ASPIRATION

Résumé—Le comportement dynamique des couches limites incompressibles laminaires et turbulentes à l'intérieur desquelles se produit une condensation a été étudié tant sur le plan expérimental que sur le plan calcul numérique. L'écoulement principal est constitué d'un mélange d'air saturé en vapeur d'eau à une température modérée (80 à 90°C), et à la pression atmosphérique. L'écart de température entre la surface d'échange et l'écoulement peut atteindre 20°C. Le flux de masse à la paroi dans un tel cas permet de déterminer le 'taux d'aspiration équivalent' dans une couche limite aspirée, pour les mêmes conditions extérieures mais sans gradient de température. La comparaison entre les deux phénomènes met en relief de nombreuses similitudes et quelques différences.

DYNAMISCHES VERHALTEN EINER GRENZSCHICHT MIT KONDENSATION AN EINER EBENEN PLATTE: VERGLEICH MIT DER ABSAUGUNG

Zusammenfassung—Das dynamische Verhalten von laminaren und turbulenten Grenzschichten, in denen Kondensationsvorgänge stattfinden, wurde experimentell und numerisch untersucht. Die Temperaturdifferenz zwischen der kalten Wand und der gesättigten Luft-Dampf-Strömung bei Atmosphärendruck und mittlerer Temperatur (80–90°C) betrug nicht mehr als 20 K. Die Berechnung der Massenstromdichte an der Wand in solch einem Fall erlaubt die Vorausberechnung der 'äquivalenten Saugrate' in einer abgesaugten Grenzschicht für dieselben Außenbedingungen, jedoch ohne jegliche Temperaturdifferenz. Der Vergleich beider Phänomene zeigt interessante Ähnlichkeiten und einige Unterschiede.

ДИНАМИЧЕСКОЕ ПОВЕДЕНИЕ ПОГРАНИЧНОГО СЛОЯ ПРИ КОНДЕНСАЦИИ НА ПЛОСКОЙ ПЛАСТИНЕ: СРАВНЕНИЕ СО СЛУЧАЕМ ВСАСЫВАНИЯ

Аннотация—Экспериментально и численно исследовано динамическое поведение ламинарного и турбулентного пограничных слоев, внутри которых имеет место конденсация. Разность температур между обтекаемой холодной стенкой и потоком насыщенной паро-воздушной смеси при атмосферном давлении и умеренной температуре (примерно 80–90°C) не превышает 20°C. Расчет потока массы на стенке для такого случая позволяет определить "эквивалентную скорость всасывания" в пограничном слое для тех же внешних условий, но при отсутствии разности температур. Сравнение двух явлений указывает на существование интересного подобия и некоторых отличий.